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Homogeneous-Heterogeneous Reactions in Carbon Nanotubes Over a Stretching Sheet

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ABSTRACT

The heat transfer and the effects of homogeneous-heterogeneous reactions on the steady boundary layer flow of carbon nanotubes (CNTs) over a stretching sheet is discussed. The governing partial differential equations and boundary conditions are transformed into a set of nonlinear ordinary differential equations by using suitable similarity transformations. There are two types of CNTs used which are SWCNTs and MWCNTs with three different types of base fluids which are water, kerosene and engine oil. These equations are then solved numerically by Maple software using shooting method. The effects of the governing parameters on the dimensionless velocity, temperature, concentration, skin friction and Nusselt numbers are analyzed and presented in graphical forms. It is found that as the homogeneous-heterogeneous reactions decreases the concentration distribution getting increase, and volume fraction parameter as well as the suction parameter will effect the fluid flow.

Keywords: Boundary layer, Heat transfer, Carbon nanotubes, Stretching sheet, Homogeneous-heterogeneous reactions.

1. Introduction

Its is found that carbon nanotubes have the remarkable thermal properties which have greater thermal conductivities, earned by its cylindrical carbon molecules origin. The range diameter of carbon nanotubes are found to be from approximately 1 to 100nm and it has length in micrometer. As reported by Hone (2004) and Antar et al. (2012), they found that for single-wall CNT and multi-wall CNT have its thermal conductivity up to 6,600 W/mK and 3,000 W/mK respectively. The researcher who started the idea to incorporate a mixture of nanoparticles and base fluid which called as nanofluid were come out from Chol and Estman (1995).

The experimental results found that for 0.01 to 0.05 volume of solid particle, the effective thermal conductivity of this mixture can be increased by almost 20 percent compared to the base fluid only, which can be referred from Eastman (1999) and Xuan and Roetzel (2000). The research on flow and heat transfer performance of nanofluids under turbulent flow in tubes conducted by Xuan and Li (2003). The experimental results found that the convective heat transfer coefficient and Nusselt number of nanofluids are enhanced by increasing the Reynolds number and also volume fraction of nanoparticles. Most of the applications of carbon nanotubes can be seen from areas like environment, energy, electronics, health care and many more, which can be refer in Ajayan and Iijima (1993) and Hofmann et al. (2007) also the application have been widely used in the past few years.

Investigation on the heat transfer behavior of CNT nanofluids have been conducted by Ding et al. (2006) with consideration for flowing through a horizontal tube, at which they discovered that the magnification of the convective heat transfer are dependent on the solid volume fraction and Reynolds number of CNTs. Then, the convective heat transfer of multi-wall carbon nanotubes based nanofluids investigated by Kamali and Binesh (2010) numerically in a straight tube under a constant wall heat flux condition. Wang et al. (2013) experimentally investigate the heat transfer and pressure drop of nanofluids in a horizontal circular tube that containing carbon nanotubes. Xue (2005) investigate the thermal conductivity of CNTs nanofluid which renovated by Yang and Xu (2017).

Chemical reactions can be divided into two types, namely homogeneous and heterogeneous reactions. The major difference that can be said from this two reaction is lie on the catalyst works individually in the same phase where reactions occurs (homogeneous) or in distinctive phase (heterogeneous). There will be no constraint on phase to be accounted. Most of the process that involve in gaseous phases are using homogeneous catalyst, while for process that using heterogeneous catalyst are in solid phase, and it whether in liquid or gaseous phase for the reactions occur. As mentioned by Shaw et al. (2013) those reactions basically take place in biochemical system, combustion, catalysis and many more. A simple model for homogeneous heterogeneous reactions in boundary layer flow from have been considered by Chaudhary and Merkin (1995b), at which it is supposed to be specified by isothermal cubic autocatalator kinetics for homogeneous (bulk) reaction while for heterogeneous in which surface reaction by first order kinetics. Chaudhary and Merkin (1995a) analyzed homogeneous-heterogeneous reactions in boundary-layer flow governed by a simple isothermal model, as inscribed by Hayat et al. (2016), in the presence of chemical reaction on an unsteady flow of couple stress fluid. Bhattacharyya et al. (2011) investigated the problem of boundary layer flow towards a stretching/shrinking sheet of stagnation point accompanied by consideration of chemical reaction.

It is found that a low thermal conductivity of heat transfer fluids have been used in microelectronics cooling, refrigeration and air-conditioning, chemical production, transportation, and many other applications. Due to this situation it does effect the system efficiency especially in terms of heat transfer characteristic. Therefore, it is necessary to enhance effective thermal conductivity of these fluids to improve heat transfer rate. One of the method that can be apply to enhance effective thermal conductivity of these heat transfer fluids, is to add nanoparticles or nanotubes in the base fluids as been mention by Chol and Estman (1995). After that many researchers have been proved experimentally for this method which can enhanced the thermal conductivity of heat transfer fluids with even small solid volume fraction of nanoparticle by Masuda et al. (1993), Mintsa et al. (2009) and others. Model from Rashidi et al. (2013) also has its significant application in heat transfer enhancement in the sense of renewable energy systems and industrial thermal management. The problem of the boundary layer flow past a stretching sheet has a lot of applications in industrial which attracted considerable attention among researchers during the past few decades. Examples of such technological process are glass-fiber, wire drawing and paper production. First investigation has been conducted to study the steady boundary layer flow on an incompressible viscous fluid over a linearly stretching plate by Crane (1970). Then the work extended by added mass transfer parameter effect on the surface which consider the suction and injection effect which can be refer from Gupta and Gupta (1977). Research by Rashidi et al. (2014) gives a great application in the study of industrial nanotechnological fabrication processes. In this paper, it discussed on wall transpiration (suction/injection) that showed to exert a substantial influence on flow characteristics. The the problem for viscous flow due to a stretching sheet with surface slip and suction studied by Wang (2009). Sankara and Watson (1985) investigated the flow of micropolar fluid past a stretching sheet.

Therefore, main objective of this paper is to investigate the effect of homogeneous heterogeneous reactions on the boundary layer flow and heat transfer characteristics over a stretching sheet of the carbon nanotubes.

2. Problem formulation

Consider the steady two-dimensional boundary layer flow over a stretching surface with heat transfer in a water/oil-based fluid containing carbon nanotubes as nanoparticle. The surface is subjected to a uniform surface heat flux. The base fluids and the CNTs are assumed to be in thermal equilibrium. It is also assumed that a simple homogeneous-heterogeneous reaction model exists as proposed by Merkin (1996) and Chaudhary and Merkin (1995a,b) of the following form:

$$A + 2B \rightarrow 3B, \quad \text{rate} = k_1 ab^2 \tag{1}$$

while on the catalyst surface we have the single isothermal first-order reaction

$$A \rightarrow B, \quad \text{rate} = k_s a \tag{2}$$

where a and b are concentrations of chemical species A and B, and k_1 and k_s are constants. This basic model reaction scheme (1) and (2) has already been examined by Chaudhary and Merkin (1995a,b) for stagnation-point boundary-layer flow. In a natural way, this scheme guarantee that the reaction rate will be zero in the external flow and thus it is zero at the outer edge of the boundary layer. Under these assumptions and boundary layer approximations, the basic equations are, see Aleng et al. (2016) and Bachok et al. (2011),

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v_{nf}\frac{\partial^2 u}{\partial y^2} \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

$$u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} = D_A\frac{\partial^2 a}{\partial y^2} - k_1 ab^2 \tag{6}$$

$$u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} = D_B\frac{\partial^2 b}{\partial y^2} + k_1 ab^2 \tag{7}$$

where x and y are Cartesian coordinates along the surface and normal to it, respectively, with u and v being the respective velocity components, T is the temperature of the nanofluid, μ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid and ρ_{nf} is the density of the nanofluid,

$$\begin{aligned} v_{nf} = \frac{\mu_{nf}}{\rho_{nf}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_{CNT}, \\ \mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \quad (\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_{CNT}, \\ \frac{k_{nf}}{k_f} = \frac{1-\varphi+2\varphi\frac{k_{CNT}}{k_{CNT}-k_f}\ln\frac{k_{CNT}+k_f}{2k_f}}{1-\varphi+2\varphi\frac{k_f}{k_{CNT}-k_f}\ln\frac{k_{CNT}+k_f}{2k_f}}. \end{aligned} \tag{8}$$

Here, φ is the nanoparticle volume fraction, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid, k_f and k_{CNT} are the thermal conductivity of the fluid and of the carbon nanotube fractions, respectively, ρ_f and ρ_{CNT} are the densities of the fluid and of the carbon nanotube fractions, respectively. D_A and D_B are the respective diffusion coefficients. Equation for $\frac{k_{nf}}{k_f}$ were taken from Xue (2005) where the model does help to determine thermal conductivity and the dimensionless heat transfer rate of nanofluid.

The boundary conditions are given by

$$\begin{aligned} u = u_w(x), \quad v = v_w, \quad T = T_w, \quad D_A\frac{\partial a}{\partial y} = k_s a, \quad D_B\frac{\partial b}{\partial y} = -k_s a \ at\ y = 0 \\ u \to 0, \quad T \to T_\infty, \quad a \to a_0, \quad b \to 0 \ as\ y \to \infty. \end{aligned} \tag{9}$$

Velocity u_w is given by

$$u_w(x) = cx, \tag{10}$$

where c is a positive constant.

We look for a similarity solution to Eqs. (3)-(7) along with the boundary condition (9) of the following transformation:

$$\begin{aligned} \eta = \left(\frac{c}{v_f}\right)^{\frac{1}{2}} y, \quad \psi = (v_f c)^{\frac{1}{2}} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \\ g(\eta) = \frac{a}{a_0}, \quad h(\eta) = \frac{b}{a_0}, \end{aligned} \tag{11}$$

where η is the similarity variable and ψ is the stream function defined as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, which similarly satisfies Equation (3). By substituting Eqs.(11) into Eqs.(4)-(7), we obtain the following ordinary differential equations:

$$\frac{1}{(1-\varphi)^{2.5}\left[1-\varphi+\varphi\frac{\rho_{CNT}}{\rho_f}\right]}f^{'''}+ff^{''}-(f^{'})^2=0 \tag{12}$$

$$\frac{\frac{k_{nf}}{k_f}}{Pr\left[1-\varphi+\varphi\frac{(\rho c_p)_{CNT}}{(\rho c_p)_f}\right]}\theta^{''}+f\theta^{'}=0 \tag{13}$$

$$\frac{1}{Sc}g^{''}+fg^{'}-Kgh^2=0 \tag{14}$$

$$\frac{\delta}{Sc}h^{''}+fh^{'}+Kgh^2=0 \tag{15}$$

subjected to the boundary conditions (9) which become

$$\begin{aligned} f(0)=S, f^{'}(0)=1,\ \ \theta(0)=1,\ \ g'(0)=K_s g(0),\ \ \delta h^{'}(0)=-K_s g(0), \\ f^{'}(\infty)\to 0,\ \ \theta(\infty)\to 0,\ \ g(\infty)\to 1,\ \ h(\infty)\to 0, \end{aligned} \tag{16}$$

where $Pr=\frac{v_f}{\alpha_f}$ is the Prandtl number, $Sc=\frac{v}{D_A}$ is the Schmidt number, $\delta=\frac{D_B}{D_A}$ is the ratio of the diffusion coefficients, $K=\frac{k_1 a_0^2}{c}$ gives a measure of the strength of the homogeneous reaction, $K_s=\frac{k_s x Re^{-\frac{1}{2}}}{D_A}$ measure of the strength of the heterogeneous (surface) reaction, $Re=\frac{cx}{v_f}$ is the Reynolds number and $S=-\frac{v_w}{\sqrt{v_f c}}$ is the mass transfer parameter.

It is expected based on most application that the diffusion coefficients of chemical species A and B to be of a comparable size. Which makes us to make further assumption that the diffusion coefficients D_A and D_B are equal, and about to take $\delta=1$. In this case, we have from (16),

$$g(\eta)+h(\eta)=1 \tag{17}$$

Thus Eqs. (14) and (15) reduce to

$$\frac{1}{Sc}g^{''}+fg^{'}-Kg(1-g)^2=0 \tag{18}$$

and are subject to the boundary conditions

$$g'(0)=K_s g(0),\quad g(\infty)\to 1. \tag{19}$$

The quantities of physical interest that involve in this paper are skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f=\frac{\tau_w}{\rho_f u_w^2},\quad Nu_x=\frac{xq_w}{k_f\left(T_w-T_\infty\right)}, \tag{20}$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0}, \tag{21}$$

where μ_{nf} and k_{nf} being the dynamic viscocity and thermal conductivity of the nanofluids, respectively. Using variable (11), we obtain

$$C_f Re_x^{\frac{1}{2}} = \frac{1}{(1-\varphi)^{2.5}} f''(0), \tag{22}$$

$$Nu_x Re_x^{-\frac{1}{2}} = -\frac{k_{nf}}{k_f} \theta'(0). \tag{23}$$

Therefore, the further section will show the numerical solution for Eqs. (12), (13) and (18) along with the boundary conditions (16) and (19) for some values of the parameters φ, S, K and K_s.

3. Numerical method

The flow and heat transfer of carbon nanotubes in three different base fluids have been investigated. The governing partial differential equations and the corresponding boundary conditions are converted into a set of nonlinear ordinary differential equations and these equations are then solved numerically by Maple software. The governing ordinary differential Eqs. (12), (13) and (18) subject to the boundary conditions (16) and (19) are solved numerically for some values of the parameters φ, S, K and K_s using the shooting method. Shooting method is known as a method for which to solve a boundary value problem by reducing it to the solution of an initial value problem. This method is very well described in the recent papers by Bachok et al. (2013) and Bhattacharyya et al. (2011). In this method, the suitable finite values of η (say η_∞) were chosen at which depend on the values of the parameters considered.

Firstly, the system of Eqs. (12), (13) and (18) is reduced to a first-order system by introducing some new variables. Then, to solve the first-order system equations as an initial value problem, it is necessary to guess missing values and apply the shooting method, then see if the guess matches the boundary conditions at the very end. To determine either the solution obtained is valid or not, it is necessary to check the velocity, temperature and the concentration profiles. The correct profiles must satisfy the boundary conditions at $\eta = \eta_\infty$ asymptotically. This procedure is repeated for other guessing values for the same values of parameters.

4. Results and discussion

Numerical solution to the set of nonlinear ordinary differential Eqs. (12), (13) and (18) with the boundary conditions (16) and (19) were obtained by using shooting method. There are three types of base fluids that been considered, namely water, kerosene and engine oil. Thermophysical properties of base fluids and carbon nanotubes are presented in Table 1. The problem for this model considered involves with four parameters, namely volume fraction parameter φ of fluid, mass transfer parameter S, strength of homogeneous reaction K and strength of heterogeneous (surface) reaction K_s.

Table 1: Thermophysical properties of carbon nanotubes and different base fluids.

Physical properties	Nanoparticles		Base fluids		
	SWCNT	MWCNT	Water	Kerosene	Engine Oil
ρ (kg/m^3)	2600	1600	997	783	824
c_p (J/kg K)	425	796	4179	2090	2340
k (W/m K)	6600	3000	0.613	0.145	0.134

The effects of the governing parameters on the dimensionless velocity, temperature, concentration, skin friction and Nusselt numbers are investigated and showed in Figures 1-10. Variation of the velocity, temperature and concentration profiles for different type of base fluids with $\varphi = 0.1$, $S = 2.5$, $Sc = 1.0$, $K = 2.0$ and $K_s = 2.0$ is shown in Figures 1, 2 and 3. Figure 1 shows the effect of different base fluid incorporate with carbon nanotubes on the dimensionless velocity. From this figure it is obvious that water has larger velocity compare to engine oil and kerosene. For Figure 2 the dimensionless surface temperature also shows that water has larger thermal boundary layer thickness compare to engine oil and kerosene. As expected, the thermal boundary layer thickness decreases with an increase in Prandtl numbers. The thermal boundary layer thickness of kerosene (Pr=21) is in between water (Pr=6.2) and engine oil (Pr=115). Same pattern happen for Figure 3 where it shows that water has larger concentration compared to engine oil and kerosene. As expected also, the concentration does related with the density of a fluid and water has larger density compared to the other base fluid. It is seen that from Figures 1-3, the velocity and concentration is larger for based fluid incorporate with MWCNTs compared to SWCNTs while different behaviour for temperature profile. The value of each base fluids density can refer from Table 1.

Figures 4, 5 and 6 represents the variations of the velocity, temperature and concentration profiles of water (Pr=6.2) for different value of mass transfer parameter S, where $S > 0$ corresponds mass suction and $S < 0$ corresponds to the mass injection. The following parameter has been fixed for Figs. 4, 5 and 6: $\varphi = 0.1$, $Sc = 1.0$, $K = 2.0$ and $K_s = 2.0$. It is seen that all the profiles satisfied asymptotically the far field boundary conditions. From Figures 4 and 5 it is clearly that the velocity and temperature profile decrease as the suction parameter increases. While for Figure 6 increasing of suction parameter cause the concentration increases as well. Inclusion of MWCNTs in base fluids shows greater velocity and concentration profile than the inclusion with SWCNTs, while different behaviour can be seen for temperature profile.

Figures 7 and 8 display the concentration profiles of kerosene (Pr=21) for different value of homogeneous-heterogeneous reactions parameter. It is seen that the concentration profiles show the same curve and satisfied asymptotically the far field boundary conditions for both figure. For Figure 7 as the value of the strength of homogeneous reaction K decreases the concentration profile getting increase and same behavior apply for Figure 8 for the value of the strength of heterogeneous reaction K_s. The higher the values of homogeneous reaction parameter K correspond to larger chemical reaction which consequently reduces the concentration distribution Hussain et al. (2018). Finally, it is seen that all profiles for Figures 1-8 satisfied asymptotically the far field boundary conditions.

Figures 9 and 10 show the variations of the skin friction coefficient and the local Nusselt number, with volume fraction parameter φ for both CNTs with three different base fluids. It can be clearly seen that the value of these quantities (the local Nusselt number and absolute value of skin friction) increase linearly with φ. Due to increase in density of CNTs base fluids from kerosene ($\rho = 783$), engine oil($\rho = 824$) to water ($\rho = 997$) with volume fraction parameter, the skin friction increases with volume fraction parameter and inclusion with SWCNTs found to be the highest skin friction compared to MWCNTs. The reduced value of thermal diffusivity leads to higher temperature gradients, hence, higher enhancement in heat transfers. From Figure 10, as volume fraction increases, the local Nusselt number getting decreasing. The water base fluid have high values of thermal diffusivity and therefore, this reduces the temperature gradients which will affect the performance of water as a working fluid. Therefore the engine oil with the smaller thermal diffusivity shows the largest value for local Nusselt number compare to kerosene and water.

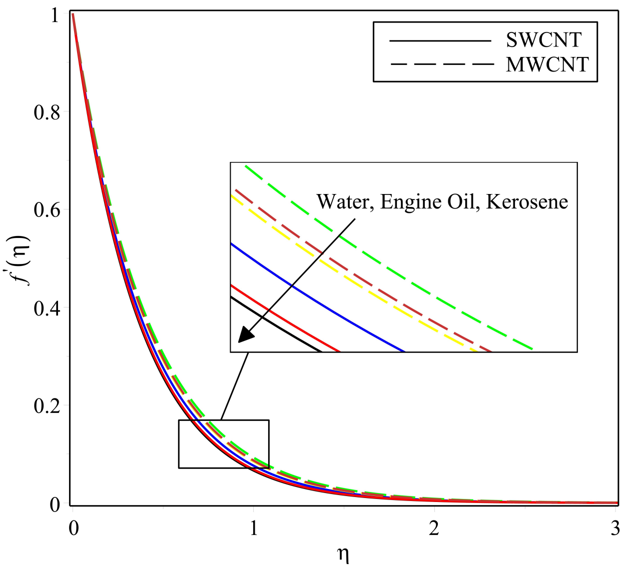

Figure 1: Velocity profiles for different base fluids

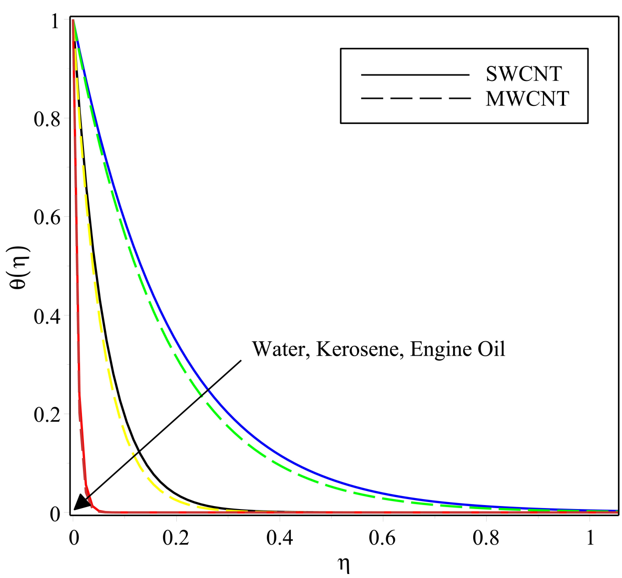

Figure 2: Temperature profiles for different base fluids

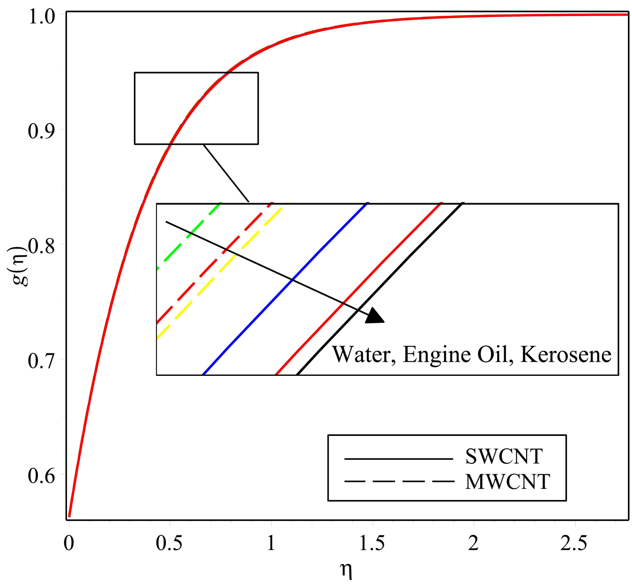

Figure 3: Concentration profiles for different base fluids

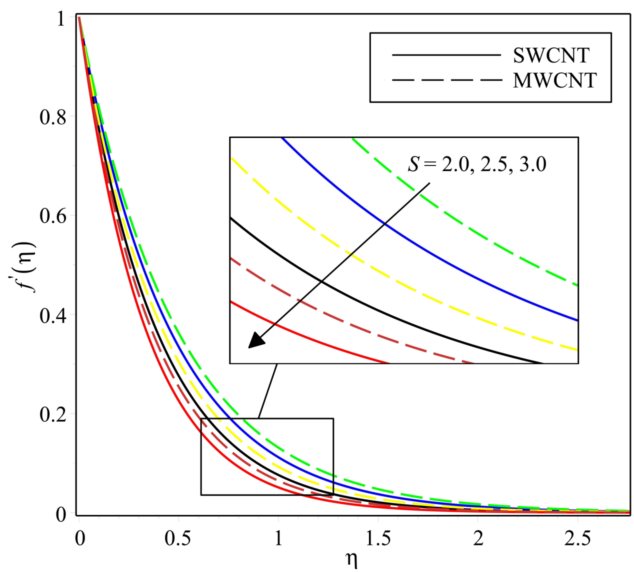

Figure 4: Velocity profiles (water) for various values of S

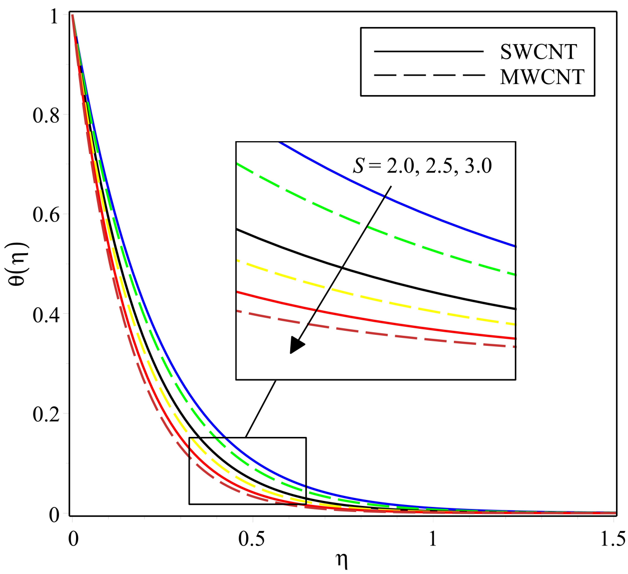

Figure 5: Temperature profiles (water) for various values of S

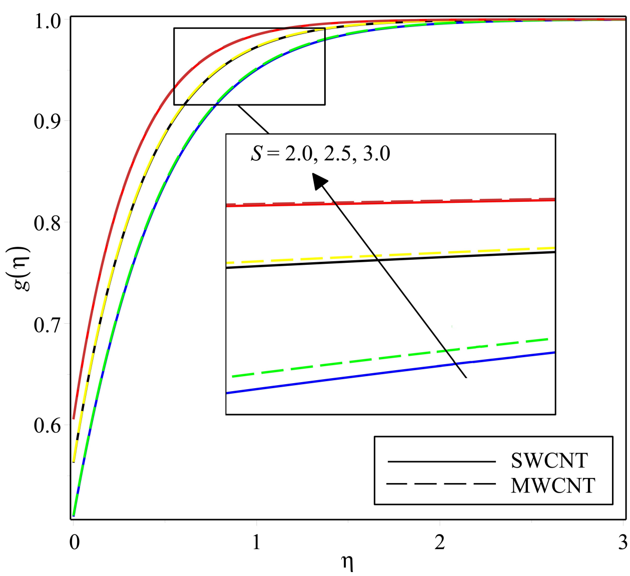

Figure 6: Concentration profiles (water) for various values of S

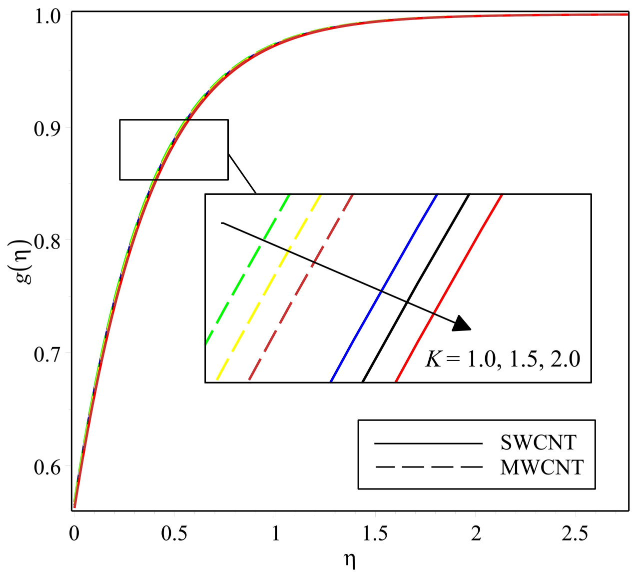

Figure 7: Concentration profiles (kerosene) for various values of K

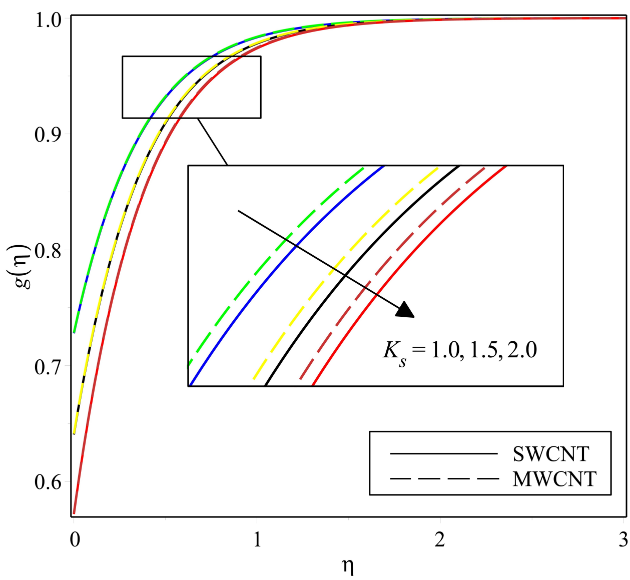

Figure 8: Concentration profiles (kerosene) for various values of K_s

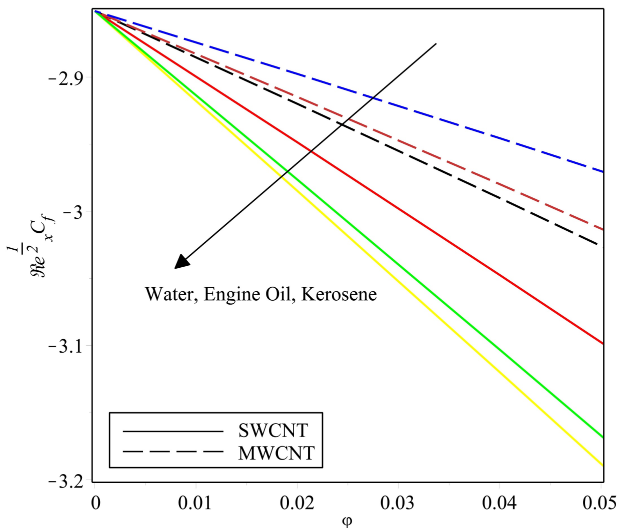

Figure 9: Skin friction coefficient with φ for different base fluid

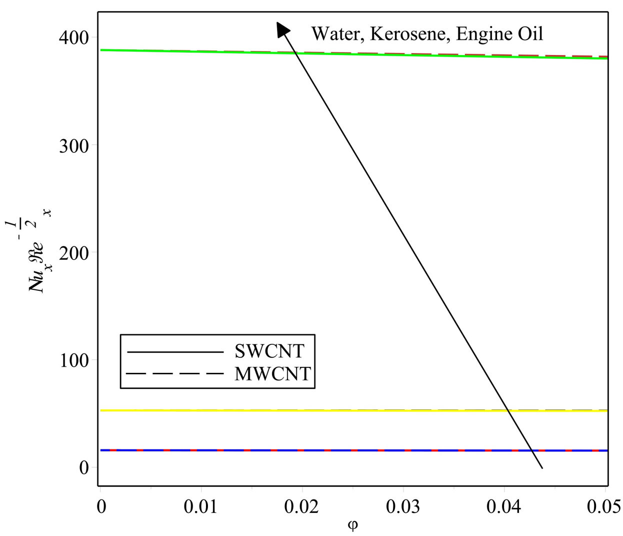

Figure 10: Local Nusselt number with φ for different base fluid

5. Conclusions

The problem of boundary layer flow and heat transfer of carbon nanotubes over a stretching sheet is studied theoretically. It is found that water has the larger velocity, temperature and concentration distribution compared to engine oil and kerosene. Decreasing of mass transfer parameter (suction $S > 0$) cause the velocity and temperature distribution increase. While directly proportional relation can be seen for mass transfer parameter and concentration distribution. It is also found that as the homogeneous and heterogeneous reactions decreases both the concentration distribution getting increase. The highest values of the skin friction coefficient and the local Nusselt number were obtained for base fluid kerosene and engine oil respectively. The inclusion of SWCNTs into base fluid increase the skin friction coefficient compared to MWCNTs inclusion and different pattern for the heat transfer coefficients.

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References

Ajayan, P. M. and Iijima, S. (1993). Capillarity-induced filling of carbon nanotubes. *Nature*, 361(6410):333–334.

Aleng, N. L., Bachok, N., and Arifin, N. M. (2016). Boundary layer flow of a nanofluid and heat transfer over a stretching shrinking sheet with suction. *Asian Journal of Mathematics and Computer Research*, 12(2):95–109.

Antar, Z., Noel, H., Feller, J. F., Glouannec, P., and Elleuch, K. (2012). Thermophysical and radiative properties of conductive biopolymer composite. In *Polymer Composite Materials: From Macro, Micro to Nanoscale*, volume 714 of *Materials Science Forum*, pages 115–122. Trans Tech Publications.

Bachok, N., Ishak, A., Nazar, R., and Senu, N. (2013). Stagnation-point flow over a permeable stretching/shrinking sheet in a copper-water nanofluid. *Boundary Value Problems*, 2013(1):39–48.

Bachok, N., Ishak, A., and Pop, I. (2011). On the stagnation-point flow towards a stretching sheet with homogeneous heterogeneous reactions effects. *Communications in Nonlinear Science and Numerical Simulation*, 16(11):4296–4302.

Bhattacharyya, Krishnendu, and Layek, G. (2011). Effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation. *International Journal of Heat and Mass Transfer*, 54(1-3):302–307.

Chaudhary, M. and Merkin, J. (1995a). A simple isothermal model for homogeneous-heterogeneous reactions in boundary-layer flow. i equal diffusivities. *Fluid dynamics research*, 16(6):311.

Chaudhary, M. and Merkin, J. (1995b). A simple isothermal model for homogeneous-heterogeneous reactions in boundary-layer flow. ii different diffusivities for reactant and autocatalyst. *Fluid dynamics research*, 16(6):335.

Chol, S. and Estman, J. (1995). Enhancing thermal conductivity of fluids with nanoparticles. *ASME-Publications-Fed*, 231:99–106.

Crane, L. J. (1970). Flow past a stretching plate. *Zeitschrift für angewandte Mathematik und Physik ZAMP*, 21(4):645–647.

Ding, Y., Alias, H., Wen, D., and Williams, R. A. (2006). Heat transfer of aqueous suspensions of carbon nanotubes (cnt nanofluids). *International Journal of Heat and Mass Transfer*, 49(1-2):240–250.

Eastman, J. (1999). Novel thermal properties of nanostructured materials. Technical report, Argonne National Lab., IL (US).

Gupta, P. and Gupta, A. (1977). Heat and mass transfer on a stretching sheet with suction or blowing. *The Canadian Journal of Chemical Engineering*, 55(6):744–746.

Hayat, T., Hussain, Z., Alsaedi, A., and Asghar, S. (2016). Carbon nanotubes effects in the stagnation point flow towards a nonlinear stretching sheet with variable thickness. *Advanced Powder Technology*, 27(4):1677–1688.

Hofmann, S., Sharma, R., Ducati, C., Du, G., Mattevi, C., Cepek, C., Cantoro, M., Pisana, S., Parvez, A., Cervantes-Sodi, F., Ferrari, A. C., Dunin-Borkowski, R., Lizzit, S., Petaccia, L., Goldoni, A., and Robertson, J. (2007). In situ observations of catalyst dynamics during surface-bound carbon nanotube nucleation. *Nano letters*, 7(3):602–608.

Hone, J. (2004). Carbon nanotubes: thermal properties. *Dekker Encyclopedia of Nanoscience and nanotechnology*, pages 603–610. New York: Marcel Dekker, Inc.

Hussain, Z., Hayat, T., Alsaedi, A., and Ahmad, B. (2018). Three-dimensional convective flow of cnts nanofluids with heat generation/absorption effect: A numerical study. *Computer Methods in Applied Mechanics and Engineering*, 329:40–54.

Kamali, R. and Binesh, A. (2010). Numerical investigation of heat transfer enhancement using carbon nanotube-based non-newtonian nanofluids. *International Communications in Heat and Mass Transfer*, 37(8):1153–1157.

Masuda, H., Ebata, A., Teramae, K., and Hishinuma, N. (1993). Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles. dispersion of al2o3, SiO2 and TiO2 ultra-fine particles. *Netsu Bussei*, 7(4):227–233.

Merkin, J. (1996). A model for isothermal homogeneous-heterogeneous reactions in boundary-layer flow. *Mathematical and Computer Modelling*, 24(8):125–136.

Mintsa, H. A., Roy, G., Nguyen, C. T., and Doucet, D. (2009). New temperature dependent thermal conductivity data for water-based nanofluids. *International Journal of Thermal Sciences*, 48(2):363–371.

Rashidi, M., Abelman, S., and Mehr, N. F. (2013). Entropy generation in steady mhd flow due to a rotating porous disk in a nanofluid. *International Journal of Heat and Mass Transfer*, 62:515–525.

Rashidi, M., Freidoonimehr, N., Hosseini, A., Bég, O. A., and Hung, T.-K. (2014). Homotopy simulation of nanofluid dynamics from a nonlinearly stretching isothermal permeable sheet with transpiration. *Meccanica*, 49(2):469–482.

Sankara, K. K. and Watson, L. T. (1985). Micropolar flow past a stretching sheet. *Zeitschrift für angewandte Mathematik und Physik*, 36(6):845–853.

Shaw, S., Kameswaran, P. K., and Sibanda, P. (2013). Homogeneous-heterogeneous reactions in micropolar fluid flow from a permeable stretching or shrinking sheet in a porous medium. *Boundary Value Problems*, 2013(1):77–86.

Wang, C. (2009). Analysis of viscous flow due to a stretching sheet with surface slip and suction. *Nonlinear Analysis: Real World Applications*, 10(1):375–380.

Wang, J., Zhu, J., Zhang, X., and Chen, Y. (2013). Heat transfer and pressure drop of nanofluids containing carbon nanotubes in laminar flows. *Experimental Thermal and Fluid Science*, 44:716–721.

Xuan, Y. and Li, Q. (2003). Investigation on convective heat transfer and flow features of nanofluids. *Journal of Heat transfer*, 125(1):151–155.

Xuan, Y. and Roetzel, W. (2000). Conceptions for heat transfer correlation of nanofluids. *International Journal of heat and Mass transfer*, 43(19):3701–3707.

Xue, Q. (2005). Model for thermal conductivity of carbon nanotube-based composites. *Physica B: Condensed Matter*, 368(1-4):302–307.

Yang, L. and Xu, X. (2017). A renovated hamilton–crosser model for the effective thermal conductivity of cnts nanofluids. *International Communications in Heat and Mass Transfer*, 81:42–50.